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Fractionally integrated ARMA for crude palm oil prices prediction: case of potentially overdifference

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Dealing with stationarity remains an unsolved problem. Some of the time series data, especially crude palm oil (CPO) prices persist towards nonstationarity in the long-run data. This dilemma forces the researchers to conduct first-order difference. The basic idea is that to obtain the stationary data that is considered as a good strategy to overcome the nonstationary counterparts. An opportune remark as it is, this proxy may lead to overdifference. The CPO prices trend elements have not been attenuated but nearly annihilated. Therefore, this paper presents the usefulness of autoregressive fractionally integrated moving average (ARFIMA) model as the solution towards the nonstationary persistency of CPO prices in the long-run data. In this study, we employed daily historical Free-on-Board CPO prices in Malaysia. A comparison was made between the ARFIMA over the existing autoregressive-integrated moving average (ARIMA) model. Here, we employed three statistical evaluation criteria in order to measure the performance of the applied models. The general conclusion that can be derived from this paper is that the usefulness of the ARFIMA model outperformed the existing ARIMA model.

Keywords: crude palm oil prices; first-order differencing; fractionally integrated; overdifference; forecast

JEL Classifications: C01; C02; C18; C53; E37

1. Introduction

One of the most significant issues amongst researchers and econometrician is the evidence of persistence towards nonstationary time series data. Relying on the assumption of the Box and Jenkins methodology, the time series is assumed to be stationary. With regard, most of the time series data are nonstationary and it is considered the norm. Where such an assumption is not met, then the necessary procedures, such as differencing ($\Delta Y_t = Y_t - Y_{t-1}$), are performed in order to achieve the stationary time series data [26]. The effort of differencing seems to be a

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good solution towards the nonstationary counterparts. An opportune remark as it is, this proxy may lead to overdifference [16]. The difference stationary is eliminating too far of the trend like characteristics and the value of the level difference indicates that there is no influence that is close to zero frequency. The power of frequency elements has not been attenuated but nearly annihilated.

Overdifference could also form unintentional issues of differencing which could significantly contribute a problem in the parameter estimation. Hurvich and Ray [20] revealed that the time series which potentially consisted of overdifference would be biased in long-memory time series analysis. As a result, the time series prediction could be insubstantial and lose its effectiveness on parameters estimation. Worse come worse, the forecasting performance one-step-ahead degrades [16]. Another important finding found by Xiu and Jin [35] is that the problem of overdifference is also accountable for the tendency of loss of important information of the time series and this problem also affects the model construction. With regard to this matter, the existing method of the autoregressive moving average (ARMA), models which is extremely important in many years, has gradually given way to the model, in this manner the autoregressive fractionally integrated moving average (ARFIMA) model has been emphasised to deal with the time series which persist towards the nonstationary time series data.

It is important to give special attention towards the ARFIMA package introduced by Doornik and Ooms [13], which has the capability to adopt the maximum likelihood estimation (MLE) to the long-memory time series data. Prior studies have noted the main weakness of adopting the MLE towards the ARFIMA estimation procedure and the problem has essentially been solved by Hosking [19] and Sowell [32]. However, Ooms and Doornik [28] list the reason why some problems remain unsolved. There will be problems in variance matrix into account which is totally inappropriate for extensions with regression parameters. This is the explanation why MLE estimation is difficult to be adopted in the ARFIMA model.

The purpose of the current study was to compare the predictability power of each forecasting techniques for ARFIMA and autoregressive-integrated moving average (ARIMA) models to predict the past historical value of crude palm oil (CPO) prices in Malaysia which consist the long memory. Additionally, this study sets out to establish the use of fractionally integrated ARMA to be an alternative in treating the time series which persist towards nonstationarity in long-memory high-frequency time series data.

The paper is organised as follows. Section 2 deals with the literature review which highlights the general reference to previous scholarly activity such as reference to current state of knowledge and research gap. Section 3 provides the material and methods. Section 4 deals with establishing the ARFIMA and ARIMA models, including the results and discussion. Finally, the last section offers the concluding explanations for this study.

2. Literature review

Most of the time series especially in the high-frequency data exhibits the long memory and it is vital to focus and further study it. The previous study reported the main limitation of existing ARMA models which are incapable to portray long-run time series data precisely [2,16,21,22,27,35]. The possible explanation for this is that the ARMA model is good in capturing short-run prediction, while ARIMA models can be better options for nonstationary series with small sample sizes [4]. Difficulties arise such as: How the large sample sizes of series indicate the nonstationarity? Is the ARIMA model prediction still reliable? With regard to this matter, the study might have been far more convincing if we adopted the ARFIMA model. The long-memory models such as ARFIMA provide better predictions especially when dealing with long-run time series data. The rudimentary knowledge of the ARFIMA model was first originated by Granger and Joyeux [18] and the extension and profound effect of this study were done by Granger [17]. However, far too little attention has been paid to ARFIMA until Baillie [5] conducted in a process the whole assessment of both the long-memory and the ARFIMA model, and this extension attracted substantial attention in econometrics time series studies.

A large number of literature deliberate the contradictions between ARFIMA and ARMA models in time series forecasting. Baillie and Chung [6] found that the ARFIMA model is superior and remarkably successful in predicting time series data compared with the ARMA model. Consistent with the previous findings, Reisen and Lopes [31] found that the ARFIMA model is efficient when compared with the ARMA model in terms of the mean square error (MSE) and outperformed up to five steps ahead. There are similarities between the characteristics expressed by ARFIMA in the study of Erfani and Samimi [16] and those described by Baillie and Chung [6] and Reisen and Lopes [31]. The application of ARFIMA has been found to be successful when compared with the ARMA model. However, it also depends on the memory parameters. The long-memory models, such as the ARFIMA, do not show adequately the mechanism that operates with short-memory parameters.

The paper of Ellis and Wilson [15] probably is the best-known critic of the ARFIMA models. They mainly argue the applicability of the ARFIMA models in conducting the out-of-sample forecasting. This might be explained by the fact that they found that the ARFIMA model produced the poor out-of-sample result, since it fails to outperform well in forecasting based from the last pragmatic value or generally known as a random walk model. At the same time, the application of ARFIMA produces high prediction variance. Therefore, they confirm that the ARFIMA model turns out to be poor out-of-sample performance. This assertion of the finding is supported by Xiu and Jin [35]. The ARFIMA model was found to be poor and ineffective in predicting the Hang Sheng Index. However, the matters of characteristics of nonlinear systems might put the time series analysis into ineffectiveness. There is a possibility if initial conditions were slightly different, which can produce completely different outcomes.

Ellis and Wilson's analysis has been criticised by a number of authors. Wang and Wu [34], for example, point out that the ARFIMA model, which takes long memory into account, can outperform based on out-of-sample forecasting. These findings are in agreement with those founded by Bhardwaj and Swanson [7] that is the ARFIMA model does not produce poor out-of-sample performance. Basically, considering the Diebold and Mariano [12] perspective about the MSE, the ARFIMA model sometimes progresses much better using out-of-sample forecast when compared with the alternative forecasting techniques. The ARFIMA model is not going to descend into the 'empty box'. With regard, Bhardwaj and Swanson [7] pointed out that the ARFIMA model generally outperforms for simple linear process at a longer prediction scope. Additionally, the matters of d are totally useful when constructing the prediction models by fine data sets such as the estimators based on minimising the loss of the predictive error.

The evidence provided by Ellis and Wilson has been devastatingly critiqued by Chortareas *et al.* [8]. They found that the out-of-sample forecasting by the ARFIMA model for high-frequency data outperforms when compared with other alternative forecasting techniques. The ARFIMA model is said to be more accurate when the time series data are observed on the basis of minutes. These findings seem to be consistent with research done by Koopman *et al.* [23], which applied the ARFIMA model to data of the S&P100 stock index, and the result of the forecast via out-of-sample indicated to be more accurate than its rivals. These findings further support the idea of Chu [9], where the ARFIMA model performs well for time series which comprises the economic and political shocks, and yet the model is found to be successful amongst the rival models during a tranquil period. Hence, the ARFIMA model is said to be better than the ARIMA model because d is treated as a noninteger number as $0.0 \leq d \leq 0.5$ when modelling nonstationary time series data. Therefore, the ARFIMA model progresses the prediction accuracy by more than a few percentage points depending on the lead which rival models are compared with.

3. Materials and methods

In this paper we use the daily CPO prices in FOB MYR/US$ per metric tonne from 1 January 2004 to 31 December 2011, where FOB (abbreviation of Free-on-Board) and MYR (abbreviation of Malaysian Ringgit) are the currency in Malaysian. This time series is presented in Figure 1.

The ARFIMA and ARIMA models were used for modelling and forecasting these data. The strategy of Box and Jenkins's modelling can be seen from Figure 2.

3.1 *Autoregressive fractionally integrated moving average*

The ARFIMA model is very useful for the time series data that have a strong persistency level towards nonstationarity [27]. The time series data Y_t considers the ARFIMA (p, d, q) model if it was stationary and fulfils noninteger value of integrated based on the following formula:

$$\Phi(L)(1-L)^d(Y_t-\mu_t)=\Theta(L)\varepsilon_t, \quad t=1,\ldots,T, \tag{1}$$

where μ_t represents the mean of Y_t, while L and ε_t are the lag operator and the white noise at time t, respectively. Also from Equation (1), the autoregressive is given by

$$\Phi(L)=(1-\phi_1L-\cdots-\phi_pL^P), \tag{2}$$

and the moving average operator is given by

$$\Theta(L)=(1+\theta_1L+\cdots+\theta_qL^q), \tag{3}$$

where p and q are integers while the d is real. The major player in this model is $(1-L)^d$ which the fractional difference operator is defined as the binomial equation (4) as follows:

$$(1-L)^d=\sum_{j=0}^{\infty}\delta_jL^j=\sum_{j=0}^{\infty}\binom{d}{j}(-L)^j. \tag{4}$$

However, in empirical studies, it is hard to adopt the ARFIMA model that allows the MLE in estimating long-memory time series data. Therefore, special attention is given towards a package introduced by Doornik and Ooms [13], whereby the ARFIMA model allows the MLE for long-memory time series data. The ARFIMA model (with $0.0 \leq d \leq 0.5$) is good to capture the time series data with persistence towards nonstationarity and has been considered by number of literature in many field of time series study. Please refer to Doornik and Ooms [13] for a complete explanation on MLE.

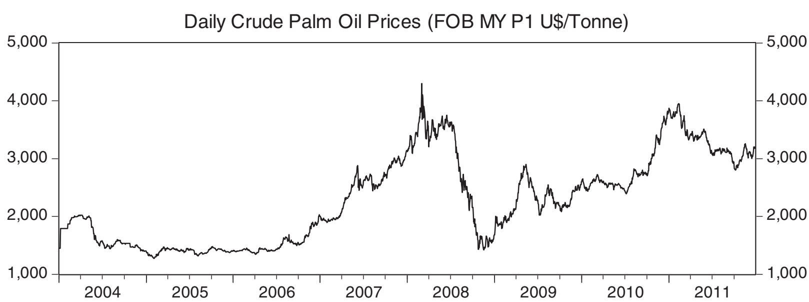

Figure 1. Plot of the original time series of CPO prices from 1 January 2004 to 30 December 2011.

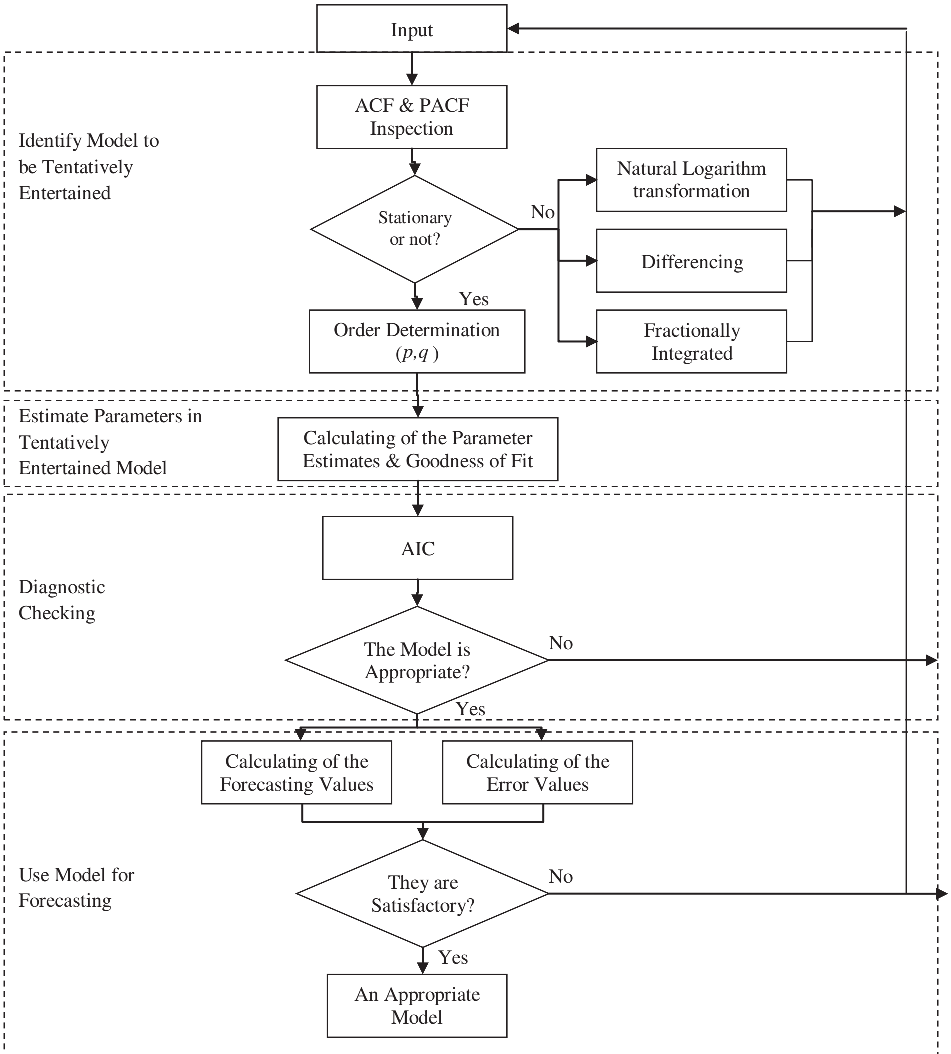

Figure 2. Flowchart of the Box and Jenkins model-building strategy.

3.2 *Autoregressive-integrated moving average*

If the original time series of CPO (denoted by Y_t) fails to meet the stationary assumption, we will consider to adopt the ARIMA model, where it is necessary to conduct one or two differences, generally. The general representation of the model is ARIMA (p, d, q), where p denotes the number of autoregressive term. Meanwhile, q represents the number of moving average term. The main player in the ARIMA model is the value of d which represents the order of the difference for the nonstationary solutions. Therefore, the general formulation of the ARIMA model can be expressed as Equation (5) as follows:

$$w_t = \mu + \phi_1 w_{t-1} + \phi_2 w_{t-2} + \cdots + \phi_p w_{t-i} - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} \cdots \theta_q \varepsilon_{t-q} + \varepsilon_t, \tag{5}$$

where, for example, w_t represents the first-order difference $(Y_t - Y_{t-1})$ of the time series data which is assumed to be stationary. The constant term is denoted by μ. The $\phi_p, i = 1, \ldots, p$, are the autoregressive coefficients to be estimated, and $w_{t-i}, i = 1, \ldots, p$, is the stationary time series

Table 1. Some descriptive statistics of the original series of the CPO prices (from 1 January 2004 to 31 December 2011).

Statistics	CPO prices
Mean	RM2284.55
Median	RM2220.96
Maximum	RM4300.67
Minimum	RM1272.50
Standard deviation	RM753.24
Skewness	0.3702
Kurtosis	1.9155

lagged by i. In addition, the terms denoted as θ_q and ε_{t-q} represent the qth coefficient of moving average to be estimated, and the qth error term, respectively. Also, the errors are uncorrelated for nonzero lags.

3.3 *Statistical evaluation criteria*

In this study, we utilised three statistical evaluation criteria in order to measure the performance of the applied models. These criteria are the root mean square error (RMSE), coefficient of determination (R^2) and Scatter Index. Their expressions are given by

$$\mathrm{RMSE} = \sqrt{\frac{\sum_t^n e_t^2}{n}}, \tag{6}$$

where e_t equals to $Y_t - \hat{Y}_t$. In particular, the terms of Y_t and $\hat{Y}_t$ present the actual observation at the point and fitted value at time t, respectively

$$R^2 = 1 - \frac{\sum e_t^2}{\sum Y_t^2}, \tag{7}$$

where $\sum Y_t^2$ is the total sum of squares and $\sum e_t^2$ is the residual sum of squares.

4. Results and discussion

The present study aims to compare the predictability power of the ARFIMA and ARIMA models to forecast daily historical CPO prices in Malaysia. Additionally, this study also sets to establish the use of fractionally integrated to be an alternative in treating the time series which persist towards the nonstationarity.

Table 1 shows some descriptive statistics of the CPO prices in Malaysia. The mean of CPO prices is equal to RM2284.55 tonne. The result of the Jarque–Bera test confirms that the null hypothesis of the normal distribution is rejected. Before we proceed to forecast daily CPO prices, it is vital to inspect the autocorrelation function (ACF) and partial autocorrelation function (PACF).

4.1 *The ACF and PACF inspections*

Figure 3 presents the results obtained from the ACF and PACF from the preliminary analysis of the CPO prices. As can be seen from Figure 3(a), we found that a covariance stationary of the CPO prices exhibits a statistically significant dependence between the observations. The illustration from the ACF inspection indicates that it decays at a hyperbolic rate (or sluggish)

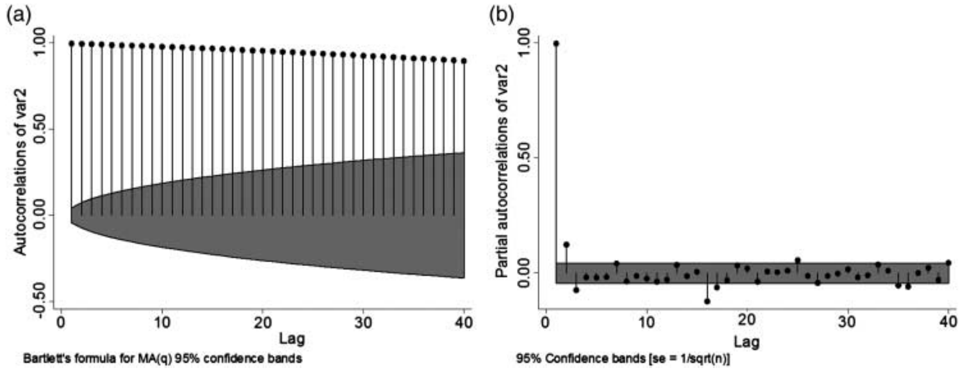

Figure 3. Plots of (a) ACF and (b) PACF of the original time series of CPO prices.

than the short-memory time series data. In this case, the time series presents the evidence of long memory [3,11,24,29,33,35].

4.2 *The unit root and stationarity tests*

The ARFIMA and ARIMA models assume that the time series is stationary. The inspection of the ACF and PACF of the CPO prices suggests a strong evidence of long memory. Therefore, it is necessary to do tests to assess such behaviour, as tests of unit root and stationarity.

There are numerous approaches to indicate the unit root and stationarity for the time series data. Thus, there are no predetermined rules as to which approach needs to be adopted in a particular condition. Therefore, for the CPO prices, we test the unit root based on augmented Dickey and Fuller [10] (ADF), Phillips and Perron [30] (PP) and Elliott *et al.* [14] Dickey Fuller test statistic using generalised least squares (DF GLS) tests, where the null hypothesis is that there is a unit root. To test the stationarity we use the Kwiatkowski, Phillips, Schmidt and Shin [25] (KPSS) test, where the null hypothesis is the stationarity around a constant.

Analysing the results of Table 2, we conclude that (a) the ADF, PP and DF GLS tests are not significant at 1%, 5% and 10% levels, and then we do not reject the null hypothesis of unit root, and (b) the KPSS test is significant at 1% level, that is, we reject the null hypothesis of stationarity at 99% confidence level.

Therefore, we consider transforming the original series of the CPO prices into a stationary series by applying both the procedures of fractionally integrated and first- (or second-) order differencing.

Table 2. Results of the tests of unit roots and stationarity for original series of the CPO prices (from 1 January 2004 to 31 December 2011).

Test	Value of statistic	1% Critical value	5% Critical value	10% Critical value
ADF	−1.882636	−3.962451	−3.411965	−3.127886
PP	−1.965649	−3.962447	−3.411963	−3.127885
KPSS	0.236970***	0.216000	0.146000	0.119000
DF GLS	−1.859572	−3.480000	−2.890000	−2.570000

Note: The critical values are based on percentage levels of 1%, 5% and 10%, which correspond to 99%, 95% and 90% of confidence level.
*Significant at levels of 10%.
**Significant at levels of 5%.
***Significant at levels of 1%.

4.3 *Establishing the ARFIMA and ARIMA models*

Relying from the fact that both of the ACF and PACF inspections and the results of the unit root and stationarity tests indicate that the original series must be transformed into stationary series about mean, calculating differences of order d with $0 \leq d \leq 0.5$ for ARFIMA models and d integer (equals 1 or 2, generally) for ARIMA models. For the ARFIMA models, we estimate the value of d through the package introduced by Doornik and Ooms [13], the result of which was equal to $d = 0.0016$, that is, $\Phi(L)(1-L)^{0.0016}(Y_t - \mu_t) = \Theta(L)\varepsilon_t$.

The resulting series is depicted in Figure 4(a), from which we can observe that there is not much loss of information from the original series. This might be explained by the fact that the fractionally integrated time series is still displaying characteristic like the trend. However, we need to be very circumspect about the stationarity of the resulting time series data after applying the fractional differencing parameter of $d = 0.0016$. With regard to this matter, it is needed to conduct the stationarity test.

Therefore, the results of the unit root and stationarity tests for the series with fractional differencing are presented in Table 3. This table is quite revealing in several ways. We found that the ADF test is significant at the 10% level, that is, with a confidence level of 90% we reject the null hypothesis that the fractionally integrated time series with $d = 0.0016$ have unit root. However, the PP test is not significant at 1%, 5% and 10% levels, and that we do not reject their null hypothesis of unit root with 99% of confidence. The DF GLS test is significant at the 5%

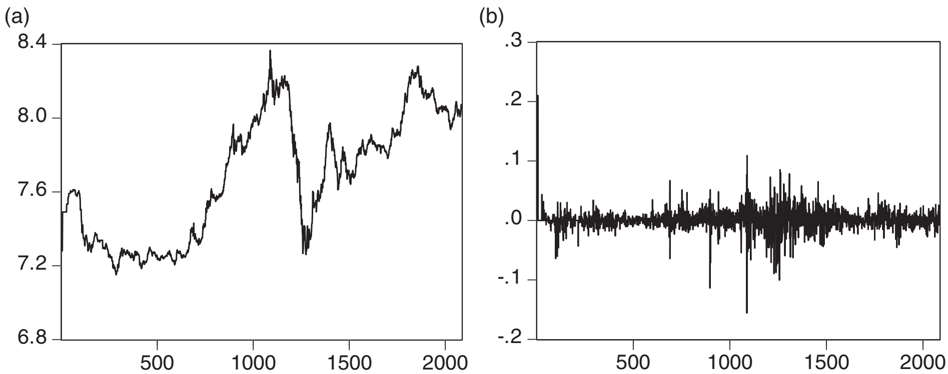

Figure 4. Plots of the (a) fractionally integrated (with $d = 0.0016$) and (b) first order difference of the original time series of CPO prices.

Table 3. Results of the tests of unit roots and stationarity for the fractionally integrated (with $d = 0.0016$) time series of the CPO prices (from 1 January 2004 to 31 December 2011).

Test	Value of statistic	1% Critical value	5% Critical value	10% Critical value
ADF	−3.392763*	−3.962620	−3.412050	−3.127940
PP	−2.471755	−3.962450	−3.411960	−3.127890
KPSS	0.133418*	0.216000	0.146000	0.119000
DF GLS	−3.247697**	−3.480000	−2.890000	−2.570000

Note: The critical values are based on percentage levels of 1%, 5% and 10%, which correspond to 99%, 95% and 90% of confidence level.
*Significant at levels of 10%.
**Significant at levels of 5%.
***Significant at levels of 1%.

level and we reject their null hypothesis of unit root with 95% of confidence. With null hypothesis that the series is stationary, the KPSS test was significant at the 10% level, and then with 90% of confidence we reject that the series is stationary. Therefore, considering these results, of the four tests, we conclude at 95% of confidence that the fractionally integrated time series with $d = 0.0016$ does not have unit root and/or is stationary.

Now for the ARIMA models, Figure 4(b) shows the series of the first-order difference ($d = 1$) of the original series of the CPO prices, which seems to have a stationary pattern but also seems too far because the characteristics of trend, for example, were not attenuated but was nearly annihilated. The series of the first-order difference indicates that the characteristic like the trend has no influence and fluctuates around the zero value. Besides, Figure 4(b) when compared with Figure 1 indicates that there is an evidence of overdifference. In this case, the resulting series differenced by $d = 1$ seems to be stationary, but it is responsible for a loss of information in daily CPO prices record.

As an effort to ensure that the time series of the first-order difference is stationary, we employed the unit root and stationarity tests whose results are illustrated in Table 4. From it, we found that the tests of ADF, PP and DF GLS are significant at the level of 1%, that is, with 99% of confidence level we reject the null hypothesis of unit root.

Turning now to the KPSS test, we see that it is not significant at 1%, 5% and 10% levels. Therefore, we do not reject the null hypothesis that the referred time series is stationary. Thus, all of the reported tests indicate that the series of first-order difference is stationary.

However, we also found that the reported results in Table 4 for the ADF, PP and DF GLS tests indicate large values of their statistics, while the KPSS shows small value of statistic. This can be explained by the possible overdifference of the original time series as illustrated in Figure 4(b).

As we have achieved, the obtained series are stationary considering both the fractionally integrated and first-order difference. Then we proceed with determining the order of p and q of the models ARFIMA and ARIMA. Prior studies have noted that it is not easy to identify precisely an appropriate order of AR and MA based on the ACF and PACF spikes [26]. Therefore, we employed the 'trial-and-error' method as one effort to reduce the risks of wrong model identification [1,21]. For this reason, we applied the Akaike's information criterion (AIC) to determine the appropriate models, and the results are summarised in Table 5. As the results show, for the ARFIMA models, we found that the ARFIMA (1,0.0016,0) is the most adequate model since the criterion is minimum for the model. Furthermore, for the ARIMA models, the ARIMA (1,1,0) shows the best criterion for the model. Interestingly, the results from Table 5 present the evidence that the two of the models have met the concept of the model simplicity (parsimony), which also points towards the model type. However, the result from the ARFIMA (1,0.0016,0) model is slightly better than that of ARIMA (1,1,0) model.

Table 4. Results of the tests of unit roots and stationarity for the first-order difference of the time series of the CPO prices (from 1 January 2004 to 31 December 2011).

Test	Value of statistic	1% Critical value	5% Critical value	10% Critical value
ADF	−30.66420***	−3.962451	−3.411965	−3.127886
PP	−48.81139***	−3.962449	−3.411964	−3.127885
KPSS	0.072322	0.216000	0.146000	0.119000
DF GLS	−30.65360***	−3.480000	−2.890000	−2.570000

Note: The critical values are based on percentage levels of 1%, 5% and 10%, which correspond to 99%, 95% and 90% of confidence level.

*Significant at levels of 10%.

**Significant at levels of 5%.

***Significant at levels of 1%.

Table 5. Results of the AIC for the ARFIMA and ARIMA models.

Model	AIC
ARFIMA (1,0.0016,0)	4.000003
ARFIMA (1,0.0016,1)	6.000003
ARFIMA (2,0.0016,0)	6.000001
ARFIMA (2,0.0016,1)	8.000001
ARFIMA (2,0.0016,2)	10.000001
ARIMA (1,1,0)	4.000671
ARIMA (1,1,1)	6.000000
ARIMA (2,1,0)	6.602374
ARIMA (2,1,1)	8.000000
ARIMA (2,1,2)	10.000000

Table 6. Performance of the forecasting errors.

Model	RMSE	R^2
ARFIMA (1,0.0016,0)	0.017034	0.997384
ARFIMA (1,0.0016,1)	0.017036	0.997383
ARFIMA (2,0.0016,0)	0.017040	0.997381
ARFIMA (2,0.0016,1)	0.017042	0.997381
ARFIMA (2,0.0016,2)	0.017182	0.997339
ARIMA (1,1,0)	0.025257	0.995126
ARIMA (1,1,1)	0.027003	0.994305
ARIMA (2,1,0)	1.064019	0.995128
ARIMA (2,1,1)	0.042292	0.984907
ARIMA (2,1,2)	0.042349	0.984852

Note: Using the Diebold and Mariano [12] prospective stat ranging (-1.2 to $+1.0$) shows that all of the reported of RMSE are statistically significant apart from for ARIMA (2,1,0).

Comparisons between the two models ARFIMA and ARIMA were made using the RMSE and R^2, and the results are illustrated in Table 6. In particular, the forecasting is then used to select the best ARFIMA model and the best ARIMA model. As we can observe, for ARFIMA models, the best fit is given by ARFIMA (1,0.0016,0), whose parameters' estimates are:

$$\Phi(L)(1-L)^{0.0016}(Y_t-\mu_t)=\Theta(L)\varepsilon_t, \tag{8}$$

with the autoregressive given by

$$\Phi(L)=1+0.9987L\,(0.0001) \tag{9}$$

and the values of RMSE and R^2 are 0.017034 and 0.997384, respectively. We found that the values of RMSE will increase and R^2 will decrease as the order of autoregressive and moving average element increases. Looking into the ARIMA model, it is clearly stated that the ARIMA (1,1,0) given by

$$w_t=0.0106-0.0013w_{t-1}\,(0.0120) \tag{10}$$

with the value of RMSE being equal to 0.025257 and being slightly better than the other ARIMA models. The values in parentheses are the p-values that indicate the ARFIMA (1,0.0016,0) and ARIMA (1,1,0) models' parameters are significant (or non-null). Overall, Table 6 is quite revealing that the ARFIMA models are superior compared with the ARIMA models.

Interestingly, the results from Table 6 present evidence that the first-order difference should be responsible for the loss of important information since it eliminates too far of the trend like characteristics which leads to the increase in errors in CPO prices forecasting. The plots of the graphs between the observed and forecasted value by ARFIMA (1,0.0016,0) and ARIMA (1,1,0) models, respectively, are shown in Figures 5 and 6. Comparing the two depicted graphs, it can be seen that the best predictions are presented by the ARFIMA model. The forecasted values from the ARFIMA model are more close to the observed CPO prices than those of the ARIMA model. The depicted graphs are consistent with the results reported in Table 6.

Further analyses on the prediction errors (e_t) generated by both of the models are displayed in Figure 7. The term (e_t) is equal to $Y_t - \hat{Y}_t$, which are the actual observation at the point and the fitted value at time t, respectively. It is apparent from graph (b) of Figure 7 that the ARIMA model shows an increasing level of noise when compared with its rival model. Consistent with the previous statistical analysis, the prediction errors prove that the ARFIMA model is the best-fit model to represent the daily CPO prices.

In Figure 8, the scatter plots were used to compare the predicted and observed CPO prices values for the models ARFIMA (1,0.0016,0) and ARIMA (1,1,0). These two graphs display the positive relationship between the forecasted and observed CPO prices. Looking at the scatter plots, we can see that both of the models present close fits towards the imaginary regression line. In particular, the ARFIMA model presents the closer fits around them than those of ARIMA model. A part of it, the ARIMA model presents more scatter compared with its rival model. The coefficient of determination is one indication that the ARFIMA model has a slightly better fit than the ARIMA model, $R^2 = 0.997$ and $R^2 = 0.995$, respectively. Therefore, considering all the numerical results, graph and analysis, we can conclude that the ARFIMA model is highly better than the ARIMA model for the case of CPO prices.

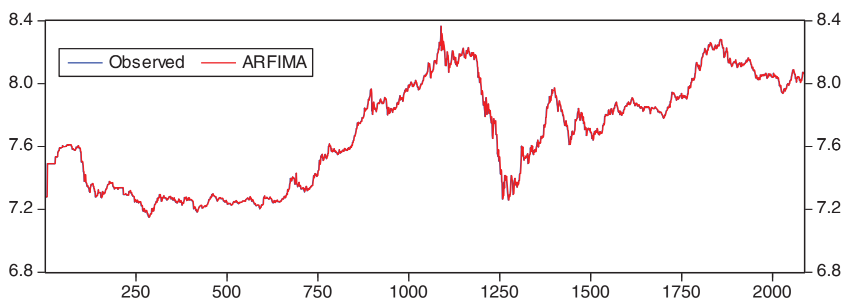

Figure 5. Plot of the original time series of CPO prices and predicted time series using ARFIMA (1,0.0016,0) model.

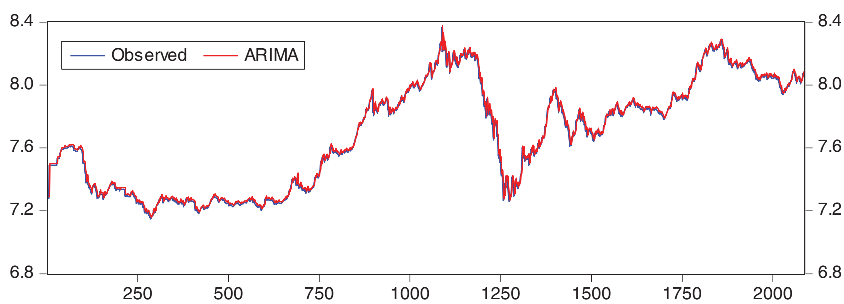

Figure 6. Plot of the original time series of CPO prices and predicted time series using the ARIMA (1,1,0) model.

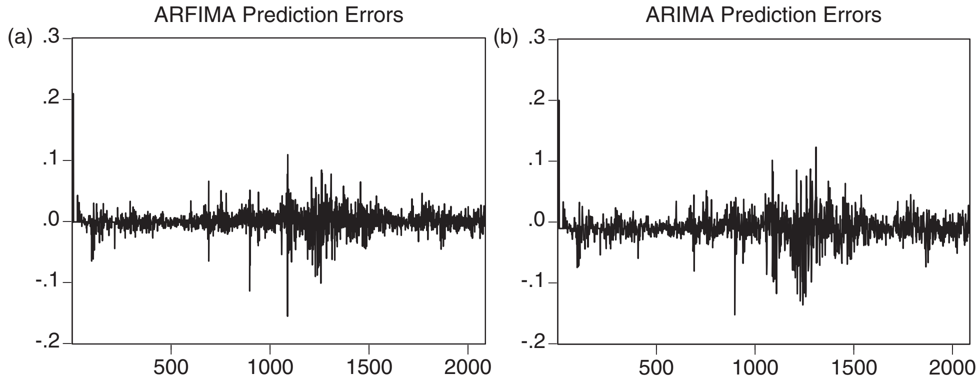

Figure 7. Plots of the prediction errors of the (a) ARFIMA (1,0.0016,0) and (b) ARIMA (1,1,0) models.

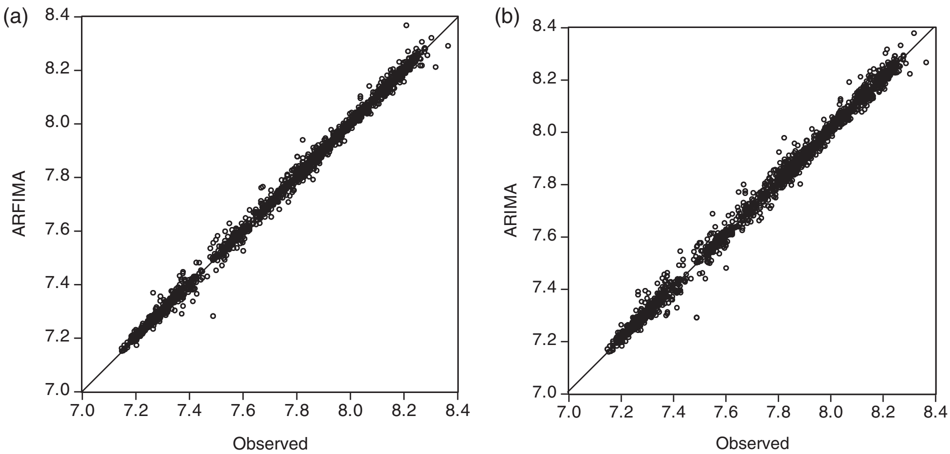

Figure 8. Scatter plots of the (a) original time series of CPO prices with predicted time series using the ARFIMA (1,0.0016,0) model and (b) original time series of CPO prices with predicted time series using the ARIMA (1,1,0) model.

5. Conclusion

The foregoing sections dealt with the time series data towards the persistence of nonstationarity. In this case, daily CPO prices in Malaysia consist of strong long memory. Therefore, the necessary procedure proposed was the fractionally integrated and also the first-order difference towards achieving adequate stationarity of the series. We found that the forecasting performance of the ARFIMA (1,0.0016,0) model produced high-quality prediction and outperformed than that of the ARIMA (1,1,0) model by considering some statistical evaluation criteria, graphics and Diebold and Mariano [12] prospective. With this, the obtained ARFIMA model, also possible due to package of MLE introduced by Doornik and Ooms [13], is superior than the adjusted ARIMA model to forecast daily CPO prices in Malaysia whose series has long memory.

In addition, from the results of the CPO prices prediction, we found that the performance of the ARIMA is not fit as the ARFIMA model. This perhaps might be explained by the fact that from an effort of detrended (or differencing) probably degrades the performance of the ARIMA model. However, it is hard to absolutely certain that the overdifference will degrade the performance of the ARIMA model, since this study just refers to one time series. Therefore, it is far more convincing to further study this by simulation.

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