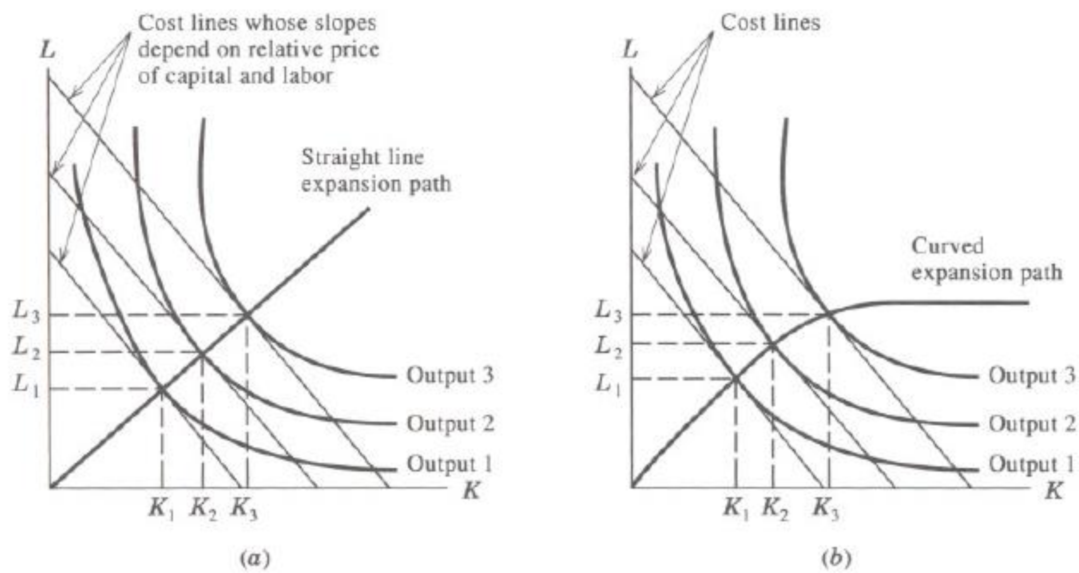
**Econ 301.01** **Bilkent University** **Berument**
**Econometrics** **Department of Economics**

**Lab Exercise 2**

Consider a production function where output (Y) depends on two inputs, capital (K) and labor *(L).* The optimal levels of *K* and *L* for a *given output* are given by the point where the "cost line" is tangential to the isoquant. The optimal levels of *K* and *L* for *different outputs* are given by the expansion path that is the locus of points where the cost lines are tangential to the isoquants. This scenario is depicted in Figure




In Figure a, the production function is such that the expansion path is a straight line; the optimal input *ratio L/K* depends on the relative input prices, but it does not depend on *Y* (it does not change as we move outward on the expansion path). In Figure *b* the production function is such that the expansion path is curved; the input ratio L/*K* depends on both the relative input prices and on output. The drawing in Figure *b* suggests L/*K* will decline as *Y* increases.

$$\frac{L_t}{K_t} = \beta_1 \left(\frac{W_t}{R_t}\right)^{\beta_2} Y_t^{\beta_3} e^{u_t}$$

Given this background, we wish to find how the labor/capital ratio depends on the relative price of labor and capital and on output for the U.S. Air Transport Industry. Observations on indices of these variables for the period 1948-79 appear in the Table. We set up the model

$$ln\left(\frac{L_t}{K_t}\right) = \beta_1 + \beta_2 ln\left(\frac{W_t}{R_t}\right) + \beta_3 \ln(Y_t) + u_t$$

*where*
$L_t$ is an index of labor input in year *t,*
$K_t$ is an index of capital services in year *t,*

$W_t$ is an index of the price of labor in year $t$,
$R_t$ is an index of the price of capital in year $t$, and
$Y_t$ is an index of output in year $t$.

1. What signs do you expect on the coefficients $\beta_2$and $\beta_3$? How does the sign on $\beta_3$ relate to the shape of the expansion path for the air transportation production function?
2. Find least squares estimates of $\beta_1$, $\beta_2$ and $\beta_3$. Interpret the estimates. Do these estimates have the expected signs?
3. Compute standard errors for the estimates of $\beta_1$, $\beta_2$ and $\beta_3$.
4. Is $\beta_3$ is different from 0 at the 5% level? Based on this, do you think the expansion path could be a straight line?
5. What proportion of variation in In $(L_t/ K_t)$ is explained by movements in In$(W_t/K_t)$ and In $(Y_t)$?
6. Test the hypothesis that $\beta_2=0$ versus and $\beta_2\neq0$.
7. Test the hypothesis that $\beta_3=2$ versus and $\beta_3\neq2$.
8. Test the hypothesis that $\beta_2+\beta_3=0$ versus not
9. Test the joint hypothesEs that $\beta_2=0$ and $\beta_3=0$. You may compare their SSRs. How????
10. Test the joint hypothesEs that $\beta_2=\beta_3=1$.
11. Test the joint hypothesis that $\beta_2=2\beta_3$.
12. Test the joint hypothesis that $\beta_2=2\beta_3 = 1$.

| Year | Y | R | K | W | L |
|---|---|---|---|---|---|
| 1948 | 1.214 | 0.1454 | 0.612 | 0.243 | 1.415 |
| 1949 | 1.354 | 0.2181 | 0.559 | 0.26 | 1.384 |
| 1950 | 1.569 | 0.3157 | 0.573 | 0.278 | 1.388 |
| 1951 | 1.948 | 0.394 | 0.564 | 0.297 | 1.55 |
| 1952 | 2.265 | 0.3559 | 0.574 | 0.31 | 1.802 |
| 1953 | 2.731 | 0.3593 | 0.711 | 0.322 | 1.926 |
| 1954 | 3.025 | 0.4025 | 0.776 | 0.335 | 1.964 |
| 1955 | 3.562 | 0.3961 | 0.827 | 0.35 | 2.116 |
| 1956 | 3.979 | 0.3822 | 0.8 | 0.361 | 2.435 |
| 1957 | 4.42 | 0.3045 | 0.921 | 0.379 | 2.707 |
| 1958 | 4.563 | 0.3284 | 1.067 | 0.391 | 2.706 |
| 1959 | 5.385 | 0.3856 | 1.083 | 0.426 | 2.846 |
| 1960 | 5.554 | 0.3193 | 1.481 | 0.441 | 3.089 |
| 1961 | 5.465 | 0.3079 | 1.736 | 0.46 | 3.122 |
| 1962 | 5.825 | 0.3783 | 1.926 | 0.485 | 3.184 |
| 1963 | 6.876 | 0.418 | 2.041 | 0.506 | 3.263 |
| 1964 | 7.823 | 0.5163 | 1.997 | 0.538 | 3.412 |
| 1965 | 9.12 | 0.5879 | 2.257 | 0.564 | 3.623 |
| 1966 | 10.512 | 0.5369 | 2.742 | 0.586 | 4.074 |
| 1967 | 13.02 | 0.4443 | 3.564 | 0.622 | 4.71 |
| 1968 | 15.261 | 0.3052 | 4.767 | 0.666 | 5.217 |
| 1969 | 16.313 | 0.2332 | 6.511 | 0.731 | 5.569 |
| 1970 | 16.002 | 0.1883 | 7.627 | 0.831 | 5.495 |
| 1971 | 15.876 | 0.2023 | 8.673 | 0.906 | 5.334 |
| 1972 | 16.662 | 0.2506 | 8.331 | 1 | 5.345 |
| 1973 | 17.014 | 0.2668 | 8.557 | 1.056 | 5.662 |
| 1974 | 19.305 | 0.2664 | 9.508 | 1.131 | 5.729 |
| 1975 | 18.721 | 0.2301 | 9.062 | 1.247 | 5.722 |
| 1976 | 19.25 | 0.3452 | 8.262 | 1.375 | 5.762 |
| 1977 | 20.647 | 0.4508 | 7.474 | 1.544 | 5.877 |
| 1978 | 22.726 | 0.5877 | 7.104 | 1.703 | 6.108 |
| 1979 | 23.619 | 0.5346 | 6.874 | 1.779 | 6.852 |