An Empirical Analysis of Istanbul Stock Exchange Sub-Indexes

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Abstract

This paper analyzes possible cointegration relations among the sub-indexes of the Istanbul Stock Exchange series - services sector, industry sector and financial sector - for the period from February 1, 1997 to September 24, 2003. The data is analyzed by using various methods initiated by Engle and Granger (1987), Johansen (1988) and Akdi (1995). The basic finding of this study is that none of these methods suggest the presence of cointegrating relationships among these indexes.
1. **Introduction**

The purpose of this paper is to examine the relationships among returns of various sub-indexes in the Istanbul Stock Exchange by using various methods. In particular, we look at the extent to which various sub-indexes are cointegrated or not by using three different methods. For the first two, Engle and Granger’s (1987) single equation models and Johansen’s (1988) multivariate cointegration methods are the among the most commonly used methods for assessing long-run relationships. Kamstra, Kramer and Levi (2003) suggest that seasonality does exist in the stock market, and addressing the seasonality in the data could alter the basic inference gathered from the data (see, Cheung and Westermann, 2003; Maravall, 1995; Hecq, 1998 and Cubadda, 1999). In order to account for this, a third method is adopted: the periodogram based cointegration procedure developed by Akdi (1995) and Akdi and Dickey (1998). This test has the advantage of being seasonality robust, and model free from the selection of the lag length. Periodogram based unitroot/cointegration tests are immune to these criticisms (see Akdi, 1995 and Akdi and Dickey, 1998).\(^1\)

The non-existence of cointegration among these sub-indexes enables the benefits of portfolio diversification among these indexed assets to be realized. However, if there is a cointegration among these indexes, then diversification will probably not lead to any benefit (see, for example, Besser and Yang; 2003 and Francis and Leachman; 1998). Most of the studies that examine the cointegration among indexes use different indexes across countries (see, Yang, Khan and Pointer; 2003 and references cited in). However, stock market indexes of different countries are subject to different monetary and fiscal policy shocks from their respective governments, as well as the specific structural problems each country may face. Thus, using data from a single country, Turkey, allows us to eliminate the effects of different policy and structural shocks on stock market indexes. Such an analysis will provide a different angle for the co-movements of the stock market indexes.

The use of data on different sectors (or sub-sectors) allows us to observe idiosyncratic elements of different sectors of the economy. In this way, we can compare different views on the source of sectoral growth. Burns and Michell (1946) argue that the broad-based swings in different sectors are driven by an unobservable aggregate cyclical component. In contrast, by using real business cycle specifications, Long and Plosser (1983) and Engle and Issler (1995) argue

\(^1\) The conventional tests (1) require estimation of too many AR parameters to address the dynamics/seasonality of the series; and (2) test results change with the sample sizes. However, the periodogram based method requires no parameter estimation except for variance (any consistent estimator of the variance can be used in the test statistics); and (3) the critical values of the test statistics are free of sample size. Therefore, especially in small samples, these might bring considerable advantages.
that the presence of sectoral components hinges on the components of sector specific shocks. Therefore, if the shocks are not common across sectors, co-movements among sectors are not likely. In particular, Durlauf (1989) argues that if “aggregate unit roots are generated by technology, it is unlikely that growth innovations will be common across sectors”. For example, improved technology in service quality in tourism may not be helpful in the home equipment sector. Stockman (1988), on the other hand, claims that both common and sector specific shocks are important for studying economic dynamics.

Data from the stock market allows us to observe the co-movements in different sectors. The stock market sub-indexes are claims on future output. These indexes could be taken as predictors of general business cycle conditions (see: Fama, 1990; Chen, 1991; and Ferson and Harvey, 1991). Thus, one may analyze the co-movements of stock market indexes to assess the role of fundamentals in various sectors.

This paper assesses the relationships among sub-indexes by using data from Turkey. Using the Turkish data has its own advantages. Turkey is an attractive emerging market for fund managers. According to the World Federation of Exchanges, for the value of share traded, the ISE is the 9th largest stock market in Europe, surpassing Ireland, Copenhagen, Oslo and Vienna. It is also the second largest emerging market after Kosdaq (Korea). Moreover, Turkey has a volatile stock market and macroeconomic performance. This high volatility allows us to minimize the type 2 error– an error made when an incorrect null hypothesis is not rejected. The basic evidence gathered from this study suggests that these sub-indexes are not cointegrated for any of the methods that were used. The following section discusses the methods, section 3 presents the empirical evidence and the last section concludes the paper.

2. Methods

Most of the statistical inference of time series is based on the stationarity assumption. The most practical way to achieve stationarity for a non-stationary series is to compute their differences. However, if a multivariate time series, $X_t$, is nonstationary, sometimes it is possible to find a vector (or matrix) $\beta$ such that $\beta' X_t$ stationary. Such a system is called cointegrated and the vector $\beta$ is called the cointegrating vector. To assess cointegration, three methods are considered in

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2 Neter, Wasserman and Kutner (1985, p71) specify that given the sample size and variance of the errors, the variance of the estimated parameters are affected by the spacing (increasing the variability of the right hand side variables) of the observed data. Thus, given the estimated parameters, increasing the spacing decreases the standard errors and increases the $t$-statistics.
this paper. The first method is the standard ordinary least squares method proposed by Engle and Granger (1987). Each component of a bivariate nonstationary time series can be written as a linear combination of two stationary and nonstationary series as

\[ X_{1,t} = a_{11}U_t + a_{12}S_t, \]
\[ X_{2,t} = a_{21}U_t + a_{22}S_t, \]  

(2.1)

where \( X_{1,t} \) and \( X_{2,t} \) are the components of a bivariate series \( X_t \). Here \( U_t \) and \( S_t \) represent unit root and stationary series, respectively. Since each component of the bivariate series includes the nonstationary component \( U_t \), both components are non-stationary. However, if the coefficients in Equation 1 are known, \( cS_t - a_{21}a_{11}S_t = cS_t \) becomes a stationary time series when the system is cointegrated. Therefore, to find any cointegrating relationship, it is enough to estimate the ratio \( a_{21}/a_{11} \). The series \( Z_t \) looks like the residuals from the regression of \( X_{2,t} \) on \( X_{1,t} \) and hence if the residual series is stationary, then the bivariate series is cointegrated.

Secondly, we will use Johansen’s trace method in order to determine if the ISE series are cointegrated or not. Consider a \( q \)th order vector autoregressive model

\[ X_t = A_1X_{t-1} + A_2X_{t-2} + \ldots + A_pX_{t-q} + \varepsilon_t, \quad t = 1, 2, 3, \ldots, T \]

then, subtracting \( X_{t-1} \) from both sides, we will have

\[ \nabla X_t = \Pi X_{t-1} + B_1 \nabla X_{t-1} + B_2 \nabla X_{t-2} + \ldots + B_q \nabla X_{t-q+1} + \varepsilon_t \]

where \( \Pi = -(I - A_1 - A_2 - \ldots - A_q) \), \( B_i = -(A_i + A_{i+1} + \ldots + A_q) \) and \( \nabla X_t = X_t - X_{t-1} \). If the matrix \( A_i, i = 1, 2, \ldots, q \) is known, then it is easy to determine the existence of any stationary linear combinations by looking at the eigenvalues of \( \Pi \) (if all eigenvalues are less than one in absolute value, then the process is stationary). However, the coefficient matrices are unknown. Let \( \text{rank}(\Pi) = r \). Johansen (1988) elaborates on a procedure to test whether there is any stationary linear combination, based on the squared canonical correlations. Let \( \lambda_i \) be the squared canonical correlations of the coefficient matrix \( \Pi \) such that \( \lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_p \) where \( p \) is the dimension of \( X_t \). The test statistic

\[ \lambda_{tr} = -n \sum_{i=r_0+1}^{p} \ln(1 - \lambda_i) \]  

(2.2)

will reject the null hypothesis \( H_0 : r = r_0 \) against the alternative of \( H_a : r \geq r_0 + 1 \) for large values of \( \lambda_{tr} \). Critical values of the distribution are given by Johansen.
(1988). That is, the null hypothesis simply says that at least \( r_0 \) linearly independent stationary linear combinations exist. If we reject the null hypothesis of \( H_0: r = 0 \) then at least one cointegrating relationship exists. Otherwise, there is no cointegration.

Finally, we will consider the estimation and testing method of cointegration based on the periodogram ordinates proposed by Akdi (1995) and Akdi and Dickey (1998). Given a time series \( X_t \), the periodogram ordinate is

\[
I_n(w_k) = \frac{n}{2} \left( a_k^2 + b_k^2 \right) \quad (2.3)
\]

where \( w_k = 2\pi k / T, k = 1,2,3,\ldots,T / 2 \) and \( a_k, b_k \) are the Fourier coefficients defined as

\[
a_k = \frac{2}{n} \sum_{t=1}^{T} (X_t - \mu) \cos(w_k t), \quad b_k = \frac{2}{n} \sum_{t=1}^{T} (X_t - \mu) \sin(w_k t). \quad (2.4)
\]

Since \( \sum_{t=1}^{T} \cos(w_k t) = \sum_{t=1}^{T} \sin(w_k t) = 0 \), the Fourier coefficients are invariant to the mean. Given a first order autoregressive time series as

\[
(\dot{Y}_t - \mu) = \rho (\dot{Y}_{t-1} - \mu) + \epsilon_t, \quad t = 1,2,3,\ldots,T \quad (2.5)
\]

the following test statistic

\[
T_k = \frac{2(1 - \cos(w_k)))}{\hat{\sigma}^2} I_n(w_k) \quad (2.6)
\]

is used to test for a unit root where \( \hat{\sigma}^2 \) is the estimated residual variance. Under the null hypothesis of a unit root (\( H_0: \rho = 1 \)), the test statistic is distributed as a mixture of chi-squares for every fixed \( k \). That is,

\[
T_k = \frac{2(1 - \cos(w_k)))}{\hat{\sigma}^2} I_n(w_k) \sim Z_1^2 + 3Z_2^2 \quad (2.7)
\]

where \( Z_1 \) and \( Z_2 \) are independent standard normal random variables. Note that the distribution is invariant to the mean. That is, the periodograms are calculated based on the original series without any model specification. Moreover, the critical values of the distribution do not depend on the sample size (see Akdi and Dickey, 1998). Since the method is based on some trigonometric transformations, the seasonality is addressed. Akdi and Dickey (1999) show that the same test statistics can be used to test for seasonal unit root. The null hypothesis of a unit root will be rejected for small values of \( T_k \). Some of the critical values of this distribution are given below (see, Akdi and Dickey, 1998):

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3 One may use a proxy of the data generating process as an any ARMA process for \( y_t \) to calculate \( \hat{\sigma}^2 \) due to Slutsky’s theorem (see Akdi, 1995: pp 33).
The periodogram analysis can also be used to estimate the cointegration vector for a multivariate time series (Akdi, 1995). Suppose that the components of a bivariate nonstationary series satisfy equation 1. As mentioned above, $Z_t = X_{2,t} - (a_{21} / a_{11})X_{1,t}$ is a stationary time series. That is, the problem is to estimate the ratio $a_{21} / a_{11}$. Let $C_k$ denote the real part of the cross periodogram ordinate of a bivariate nonstationary time series and $V_k$ be the periodogram ordinate of one of the component of a bivariate series (say the first component). Then consider a regression of $C_k$ on $V_k$. In other words, the model is

$$C_k = \alpha + \beta V_k + \eta_k, \quad k = 1, 2, 3, \ldots, [T/2]$$

(2.8)

where $[T/2]$ denotes the integer part of $T/2$. According to model (2.8), the ordinary least squares estimator of $\beta$ is a consistent estimator for the ratio. That is, as $T \to \infty$

$$\hat{\beta}_T = \frac{\sum_{k=1}^{[T/2]} (C_k - \bar{C})(V_k - \bar{V})}{\sum_{k=1}^{[T/2]} (V_k - \bar{V})^2} \to a_{21} / a_{11}$$

(2.9)

where $\bar{C}$ and $\bar{V}$ are the means for $C_k$ and $V_k$ respectively. Therefore, the cointegrating vector would be $(\hat{\beta}_T, 1)$.

### 3. Empirical Evidence

#### (a) Identification

Figure 1 plots the daily ISE series in the logarithmic form for the sample from February 1, 1997 to September 24, 2003. The components of the ISE series, $X_s = \log(\text{services})$, $X_f = \log(\text{finance})$, $X_i = \log(\text{industry})$, are modeled as a $q$th order autoregressive time series. Visual inspection suggests that

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>0.01</th>
<th>0.025</th>
<th>0.05</th>
<th>0.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_k(\alpha)$</td>
<td>0.0348</td>
<td>0.088</td>
<td>0.178</td>
<td>0.368</td>
</tr>
</tbody>
</table>

4 The data was obtained from the Central Bank of the Republic of Turkey’s data delivery system.
the ISE series may have drifts. Therefore, the models considered for these series are:

\[ X_{i,t} = \alpha_0 + \alpha_{trend} t + \alpha_1 X_{i,t-1} + \alpha_2 X_{i,t-2} + ... + \alpha_q X_{i,t-q} + e_{i,t} \]

\[ t = 1,2,3,...,T, i = S, F, I \]

(b) Unit Root Analysis

All three series are analyzed in order to see whether they include a unit root or not by using the Augmented Dickey-Fuller (with a constant term and time trend, and a constant term without time trend) periodogram based unit root tests. In order to apply the Augmented Dickey-Fuller method, we regress \( \nabla X_{i,t} \), \( i = S, F, \) and \( I \) on a constant, time trend (when applicable), \( X_{i,t-1} \) and \( \nabla X_{i,t+1-j} \), \( j = 2,3,...,q \) where \( q \) is determined by the longest significant lag rule for each variable as suggested by Ng and Perron (1995). Later we tested whether the regression coefficient of \( X_{i,t-1} \) is zero or not. That is, we considered the model

\[ \nabla X_{i,t} = \alpha_0 + \alpha_2 t + \varphi_1 X_{i,t-1} + \sum_{j=2}^{q} \varphi_j \nabla X_{i,t+1-j} + e_{i,t} \] (Enders, 1995, pp. 233)

and the value of the \( \hat{\tau} \) statistic is calculated for each series where \( \hat{\tau} = \frac{\hat{\varphi}_1 - 1}{s(\hat{\varphi}_1)} \). If the value of this statistic is less than 5% critical value (-3.415), then we reject the null hypothesis of unit root.

In order to apply the periodogram based unit test procedure, we calculate the value of \( T_k \) for each series and if the value is less than 5% critical value (0.178), then we cannot reject the null hypothesis of a unit root. The test statistic is consistent for each \( k \) and it is suggested that the low frequencies be used. Therefore, in the unit root analysis \( T_1 \) was used instead of \( T_k \). The results are given in the following table.

| Series | Panel A: Level | | Panel B: First Differences |
|--------|---------------||---------------------------|
|        | Constant     | Trend | Periodogram | Constant | Trend | Periodogram |
| \( X_S(13) \) | -2.568 | -2.237 | 3.60227 | -10.029* | -10.113* | 0.00031* |
| \( X_F(15) \) | -2.078 | -2.158 | 6.82261 | -8.867* | -8.892* | 0.00004* |
| \( X_I(17) \) | -1.221 | -2.180 | 8.16783 | -8.590* | -8.590* | 0.00003* |

* indicates the rejection of null at 1% level

Table 1 reports the unit root tests for the series. The first column reports the name of the series and the lag length in parentheses (as suggested by Ng and Perron, 1995) for the ADF tests. The second column reports the ADF tests with constant term. We clearly cannot reject the unit root for either of the series. Column 3 includes time trend and the constant term for the ADF series, column 4 reports the unit roots by using the periodogram based tests. Neither of these tests could reject the unit root in either of the series. We also repeat the analysis for the first difference of these series in Panel B. This time, we reject the null of unit root. Thus, we claim that all 3 series are I(1).

(c) Cointegration Analysis

In the previous sub-section, it was determined that all three series are first order integrated time series. Therefore, we searched for a possible cointegrating relationship among these components. We will use three different approaches, Engle and Granger (1987), Johansen (1988) and the periodogram based analysis in order to find such a cointegrating relationship.

C1) Engle and Granger (1987)'s two-step method: In this part, the components of the ISE series are investigated to determine the existence of any bivariate cointegration. The possible linear combinations are: $X_S$ vs $X_F$, $X_S$ vs $X_I$ and $X_F$ vs $X_I$.

Regress $X_F$ on $X_S$. That is, we consider the regression model as 

$$
X_{F,t} = \beta_{1,0} + \beta_{1,1} X_{S,t} + e_{1,t}, \quad t = 1, 2, 3, ..., T
$$

(3.1)

The estimated model is $\hat{X}_{F,t} = -2.054 + 1.286 X_{S,t}$. Now, consider the residual series $\hat{e}_{1,t} = X_{F,t} - \hat{X}_{F,t}$ and if this residual series is stationary, then these two series are cointegrated. Now, regress $\nabla e_t$ on $e_{t-1}$,

$$
\nabla \hat{e}_{1,t} = \alpha_{1,1} \hat{e}_{1,t-1} + \eta_{1,t}
$$

(3.2)

If we reject the null hypothesis $H_0 : \alpha_{1,1} = 0$, the series $X_S$ vs $X_F$ are cointegrated. The results for this relationship and others are given in Table 2(a). The first column reports the name of the bivariate variables. The second column is for the estimated parameter for the independent variable in equation (3.1). The third column reports the t-statistics for the $\alpha_{1,1}$ estimation in equation (3.2).
Table 2(a): Engle and Granger Method for Cointegration

<table>
<thead>
<tr>
<th>Relation</th>
<th>$\hat{\beta}_1$</th>
<th>$\hat{\epsilon}(E - G)$</th>
<th>5% Critical Value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_F \ vs \ X_S$</td>
<td>1.286</td>
<td>-1.626</td>
<td>-1.94</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
<tr>
<td>$X_I \ vs \ X_S$</td>
<td>1.082</td>
<td>-0.529</td>
<td>-1.94</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
<tr>
<td>$X_I \ vs \ X_F$</td>
<td>0.879</td>
<td>-1.712</td>
<td>-1.94</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
</tbody>
</table>

Following Table 2(a), we fail to reject the null of no-bivariate cointegration for any of the bivariate relationships.

C2) Johansen’s Method: In order to perform this analysis, we consider a 3-variate series such as $X_t = (X_{S,t}, X_{F,t}, X_{I,t})'$ and the corresponding squared canonical correlations based on 1663 observations are

$\lambda_1 = 0.01164, \quad \lambda_2 = 0.003944, \quad \lambda_3 = 0.001086$

Now, consider testing the null hypothesis of no cointegration against the alternative of at least one cointegrating relationship. That is, we consider the following hypothesis testing problem

$H_0: r = 0 \ vs \ H_a: r \geq 1.$

The value of Johansen’s trace statistic is

$\lambda_r = -T \sum_{i=1}^{3} \ln(1 - \lambda_i) = 27.049$

which is smaller than the 5% critical value (with $m = p - r_0 = 3$, Table 1 of Johansen, 1988) 31.26 and therefore, we fail to reject the null hypothesis of no cointegration.

C3 The Periodogram Method: There are 3 possible bivariate relationships.
(i) First, we will look at the relationship between $X_F$ and $X_S$. We calculate the real part of the cross periodogram ordinate of $X_F$ and $X_S$ (say $C_k$) and the periodogram ordinate of $X_S$ (say $V_k$) and regress $C_k$ on $V_k$. That is, we consider the model,

$C_k = \alpha_{1,0} + \beta \tilde{V}_k + \eta_{1,k}, \ k = 1, 2, \ldots, [T / 2]$

The OLS estimator of $\beta$, say $\hat{\beta}^{p}_1$, is a consistent estimator for the ratio given above (see Akdi, 1995). Therefore, if the series $Z_t = X_F - \hat{\beta}^{p}_1 X_S$ is stationary, then these two series are cointegrated. The results for this relationship and others are given in Table 2(b).
According to Table 2(b), we fail to reject the null of no-bivariate cointegration for any of the bivariate relationships.\footnote{The critical values are tabulated for different sample sizes and different significance levels with 10000 replicates.}

Overall, we could not find any cointegration or long-run relationship among those indexes. However, this does not mean that there is no relationship among those indexes in any time frame. Thus, we also explore the possibilities of the short-run relationship among those indexes by calculating the correlation coefficients among the growth rates of each index. Table 3 suggests the presence of positive strong correlations among those indexes. Since there is no benefit of diversification under a correlation coefficient of 1, we test if the correlation coefficient is one. If one approximates the standard errors of the correlation coefficient with $1/\sqrt{T}$, we cannot reject these coefficients different from 1 for any bivariate relationship. There are (although limited) benefits of portfolio diversification. Therefore, we conclude that there is a benefit in the long run as well as the short run of diversifications of portfolios.

<table>
<thead>
<tr>
<th>Relation</th>
<th>$\hat{\beta}_i$</th>
<th>$\hat{f}(a)$</th>
<th>5% Critical Value</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_F$ vs $X_S$</td>
<td>1.40003</td>
<td>-1.820</td>
<td>-3.43564</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
<tr>
<td>$X_I$ vs $X_S$</td>
<td>1.10583</td>
<td>-0.612</td>
<td>-3.43564</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
<tr>
<td>$X_I$ vs $X_F$</td>
<td>0.86281</td>
<td>-1.567</td>
<td>-3.43564</td>
<td>Fails to reject the null hypothesis of no cointegration</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Service</th>
<th>Finance</th>
<th>Industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Service</td>
<td>1.000000</td>
<td>0.838365</td>
</tr>
<tr>
<td>Finance</td>
<td>0.838365</td>
<td>1.000000</td>
</tr>
<tr>
<td>Industry</td>
<td>0.880866</td>
<td>0.904713</td>
</tr>
</tbody>
</table>

Critical Values for $\tau_\rho$

<table>
<thead>
<tr>
<th>N</th>
<th>%1</th>
<th>%5</th>
<th>%10</th>
<th>%90</th>
<th>%95</th>
<th>%99</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>-4.01851</td>
<td>-3.43564</td>
<td>-3.12867</td>
<td>-1.06775</td>
<td>-0.69079</td>
<td>0.08872</td>
</tr>
<tr>
<td>1600</td>
<td>-3.91286</td>
<td>-3.41016</td>
<td>-3.12126</td>
<td>-1.02745</td>
<td>-0.6483</td>
<td>0.08799</td>
</tr>
</tbody>
</table>
4. Conclusion

This paper assesses whether there is any long-run relationship among sub-indexes of the ISE by using three different cointegration methods. The empirical evidence gathered here could not find any long-run relationships among these indexes. In particular, there exists no bivariate cointegration relationship among the components of the ISE series when we use an Engle-Granger regression method and the periodogram based test. Moreover, we were unable to find any cointegrating relationships among these three indexes when the Johansen’s trace method was used.

**Figure 1:** Graphs of the Series

<table>
<thead>
<tr>
<th>$X_S$ = log(Services)</th>
<th>$X_F$ = log(Finance)</th>
<th>$X_I$ = log(Industry)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td><img src="image2.png" alt="Graph 2" /></td>
<td><img src="image3.png" alt="Graph 3" /></td>
</tr>
</tbody>
</table>
References:


